Simulation of Reflectometry in Toroidal Plasmas

E. Valeo, in collaboration with G. Kramer, N. Bertelli,
E. Feibush, R. Nazikian, B. Tobias, and A. Zolfagahari

Research and Review Seminar, PPPL, June 12, 2015

What is reflectometry?

- Microwaves are launched (nearly) radially inward, usually from the low field side, at frequencies below the maximum cutoff frequency
- The complex reflected field amplitude amplitude is anaylzed to infer
  - Plasma profile (group delay)
  - Characteristics of density fluctuations. $\Leftarrow$
- Review articles E. Mazzucato, RSI, **69**, 2201 (1998) [**1**], and also R. Nazikian *etal*, Phys. Plasmas, **8**, 1040 (2001) [**2**]

Transmitter launches electron waves

Essentially cold plasma waves (but not for center of ITER, JET, TFTR) nearly perpendicular propagation, near plasma midplane.

$$n_\perp^2 = \varepsilon(x,\omega) = \begin{cases} \frac{RL}{S} & = 1 - \frac{X}{1-\frac{Y^2}{1-X}} \quad \text{“X”} \\ P & = 1 - X \quad \text{“O”} \end{cases}$$

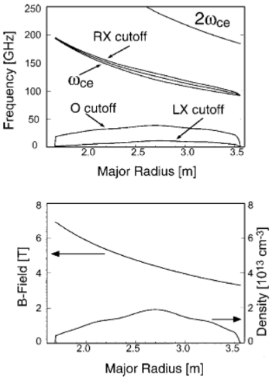
where $X = (\omega_p/\omega)^2,\ Y = \omega_c/\omega$.

Reflection at

$$P = 0 \quad \rightarrow \quad \omega = \omega_p$$

$$R = 0 \quad \rightarrow \quad \omega = (\frac{\omega_c^2}{4} + \omega_p^2)^{1/2} + \frac{\omega_c}{2}$$

$$L = 0 \quad \rightarrow \quad \omega = (\frac{\omega_c^2}{4} + \omega_p^2)^{1/2} - \frac{\omega_c}{2}$$



TFTR Profiles, from [1]

In 1D a density fluctuation produces a phase shift

$$\left(\frac{d}{dz^2} + k_0^2 \varepsilon\right) E = 0$$

In the geometrical optics limit $(k_0 \varepsilon)^{-1} d\varepsilon/dz << 1$ the solution is

$$E(z) \sim \frac{1}{k^{1/2}} \left[\exp(i\phi) + R \exp(-i\phi)\right]$$

where

$$\phi = \int_{z_c}^{z} dz' k(z') \text{ with } k = k_0\, \varepsilon(z,\omega)^{1/2} \text{ and } \varepsilon(z_c,\omega) = 0\,.$$

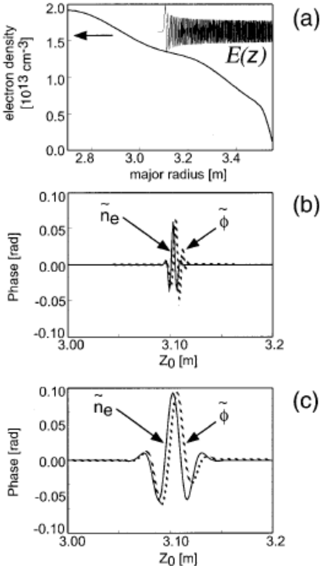
If $n = n_0(z) + \delta n$, and, correspondingly, $\phi = \phi_0 + \delta\phi$, then

$$\delta\phi = 2 \int_{z_{antenna}}^{z_c} \frac{\partial k}{\partial n} \delta n\, dz = 2k_0 \int_{z_{antenna}}^{z_c} \frac{\delta n}{\sqrt{\varepsilon}} \frac{\partial \varepsilon}{\partial n}\, dz$$

No amplitude fluctuations: $\delta R = 0$.

1D Full Wave solution confirms sensitivity of $\delta\phi$ to δn near cutoff

N. Bretz, Phys Fluids B (1992) [**3**]



115 GHz X-mode outer midplane launch, TFTR profile ($k_0 = 24$ cm^{-1})

$$\tilde{n}_e = 10^{-3} n_0 \exp[-\frac{(z-z_0)^2}{W}] \cos[k_z(z-z_0)]$$

(b) $W = .5 W_{Airy}$ $k_z = 8$ cm^{-1}
(c) $W = 10 W_{Airy}$ $k_z = 2$ cm^{-1}

where $W_{Airy} = 0.5 L_\varepsilon^{1/3} \lambda_0^{2/3} \simeq$ width of last fringe of FW solution

Two statistical quantities are measured

If $E(\omega)$ is the complex reflected signal at the receiver, then, both the coherent amplitudes

$$G(\omega) = \frac{|\langle E(\omega)\rangle|}{\sqrt{\langle |E|^2\rangle}}$$

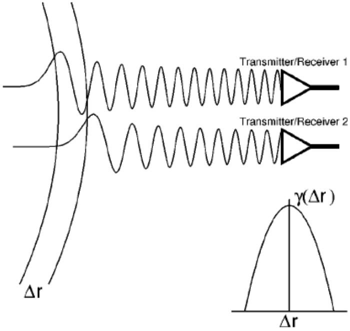
and the cross-correlations,

$$\gamma(\omega,\omega') = \frac{< |E(\omega)E^*(\omega') >}{\sqrt{< |E(\omega)|^2 >< |E(\omega')|^2 >}}$$

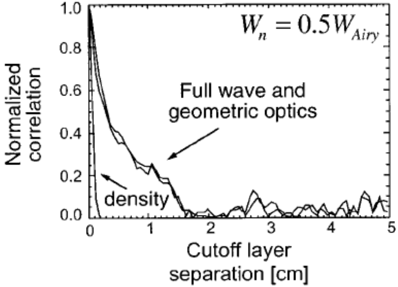
are available. $G(\omega)$ decreases with increasing fluctuation amplitude. The variation of γ vs the separation of the reflecting surfaces

$$\Delta r \simeq (\omega - \omega')\frac{\partial r_c}{\partial \omega}$$

is used to infer the turbulent correlation length.



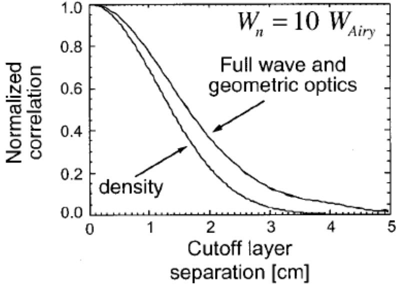
1D Simulations demonstrate reflectometer resolution somewhat greater than Airy width [ref 2]

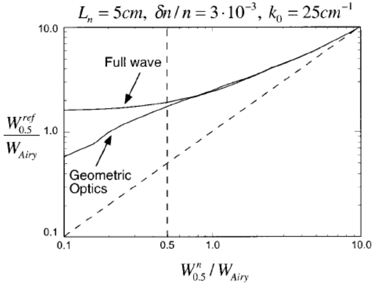


$$\frac{\langle n(r)n(r+\Delta r)\rangle}{n^2} = (\frac{\delta n}{n})^2 C_n$$

where

$$C_n = exp[-(\Delta r/W_n)^2]\cos(k_r \Delta r)$$





In higher dimensionality, multiple competing effects become important

- Curvature of the reflecting surface
- Refraction
- Diffraction
- Spatially dependent magnetic field orientation $\mathbf{b}(\mathbf{r})$
- Antenna orientation and gain pattern

FDTD propagation codes have been developed to analyze these effects.

- 2D codes
  - Y. Lin, *et al*, Plasma Phys. Control. Fusion **43** L1, (2001)
  - J. C. Hilesheim, *et al*, Rev. Sci. Instr. **83** 10E331 (2012)
  - E. Blanco and T. Estrada, Plasma Phys. Control. Fusion **55**, 125006 (2013)
  - C. Lechte, IEEE Trans. Plasma Science **37**, 1099 (2009)
- 3D codes
  - S. Hacquin, *et al*, Journees scientifiques (2013)
  - K. S. Reuther, *et al*, APS, DPP Abstract JP8.021 (2013)

Computational demands are extensive

- Wavelength is much less than machine size $S = R/\lambda \sim 10^2 - 10^3$
- Resolution requires $N \sim 20\,S$ mesh points in each dimension.
- To satisfy Courant condition, operations scale as N^{D+1} in D dimensions
- The objective is usually to construct a "synthetic" reflectometer signal by computing outgoing radiation for each realization of $\delta n(\mathbf{r})$ selected from an ensemble
- In order attain acceptable variance, at least hundreds of realizations are computed

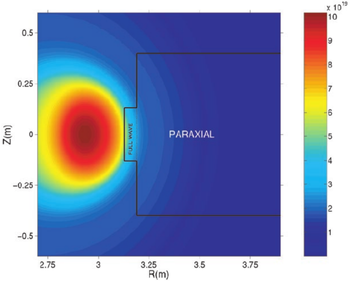
A multiple region model, FWR2D, was developed to lower the computational requirements

Assume nearly radial propagation

$\mathcal{E}(\vec{x}, t) = \Re[\exp(-i\omega t) E(\vec{x}, t)]$ and assume $\partial E/\partial t << \omega E$

$$2i\omega \frac{\partial E}{\partial t} + [c^2\nabla^2 + \omega^2 \varepsilon(R, z)]E = 0$$

is solved near the reflection layer.



Away from the reflection layer, the steady state paraxial approximation

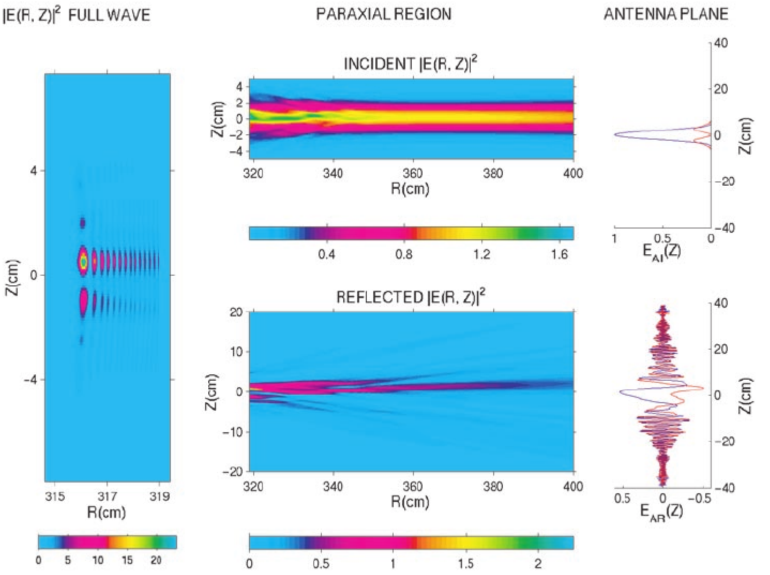
$$E(\vec{x}, t) = E_{PI} \exp(-i\phi) + E_{PR} \exp(i\phi)$$

with $\phi = \int^R dR\ k_R$, where $k_R = k_0\sqrt{\varepsilon(R, Z_0)}$, produces

$$\pm 2i \frac{\partial}{\partial R} k_R^{1/2} E_P + \frac{\partial^2}{\partial Z^2} k_R^{1/2} E_P + [\varepsilon(R, Z) - \varepsilon(R, Z_0)] k_R^{1/2} E_P = 0\,,$$

The FW and paraxial solutions are matched on a line $R = \mathrm{const}$.

For each realization of $\delta n(x, z)$ the complex reflected field $E(z)$ is computed



FWR2D has been used both to help interpret data and to optimize exprimental configurations

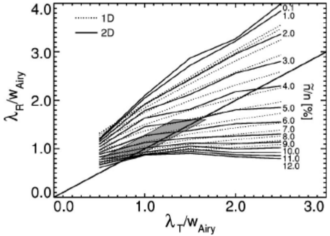
- Compare with probe and reflectometry data from LAPD
  [G. J. Kramer, *et al*, Rev. Sci. Instr. **74**, 1421 (2003)]
- Evidence for reduction of turbulent correlation length associated with transport barrier in JT-60
  [R. Nazikian, *etal* Phys. Rev. Letters **94** 135002 (2005)]
- Compare synthetic with optical imaging capabilities
  [G. J. Kramer, *et al* Plasma Phys. Control Fusion **46** 695 (2004)]
- Assess importance of T_e dependence of cutoff layer position for ITER reflectometry.
  [G. J. Kramer, *et al*, Nuc. Fusion **46** S846 (2006)]
- Aiding design of imaging reflectometer experiments on DIII-D
  [X. Ren, *et al*, Rev. Sci. Instr. **83**, 10E338 (2012); *ibid* **85** 11D863 (2014)]
- Help interpet edge reflectometer measurements on NSTX
  [A. Diallo, *et al*, Phys Plasmas **20** 012505 (2013)]

Results for G and λ_R from 1 and 2 D simulations were compared with data from LAPD

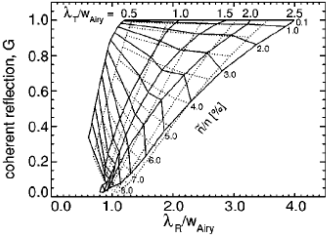
- M. Gilmore, W. A. Peebles and X. V. Nguyen, Plasma Phys. Cntrolled Fusion **42**, L1 (2000)
- $R = 60$ cm, $R_c = 50$ cm, $L_n = 10$ cm, O mode at 12 GHz
- Probes measured λ_T and δn
- Reflectometer measured G and λ_R
- Computational statistics are accumulated over many (6000 in 1D, 300 in 2D) realizations, where

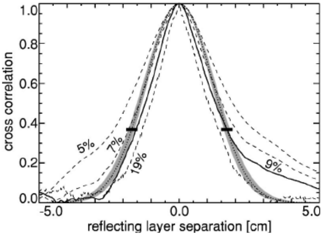
$$\frac{\langle \tilde{n}(x_1)\tilde{n}(x_1+\Delta x)\rangle}{n^2} = \left(\frac{\tilde{n}}{n}\right)^2 \exp\left[-\left(\frac{\Delta x}{\lambda_T}\right)^2\right] \cos[k_{fl}\Delta x]$$

- λ_T is not equal to λ_R
- The relationship varies with δn



Simulations map $(\delta n/n, \lambda_T) \Rightarrow (G, \lambda_R)$

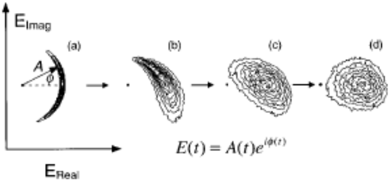




Simulations using a probe measured $\lambda_T = 1.7$ cm, were done for different δn. The computed and measured cross-correlation agreed best for the probe measured $\delta n = 9\%$.

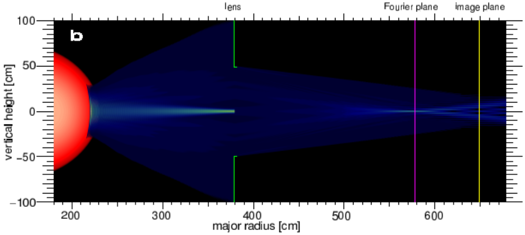
Basis for imaging

- y dependent phase variations are impressed at the reflection surface x_c
- With increasing distance from x_c, amplitude variations arise because of interference

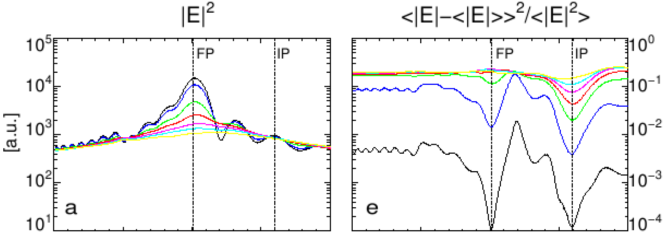


- It may be possible to "unwind" the propagation with a lens and remove the amplitude fluctuations on an image plane. [E. Mazzucato, Nucl. Fusion **41**, 203 (2001)]
- Alternatively, imaging of a rigidly convecting pattern may be possible with a single detector by numerically "unwinding" the phase (back projection). [R. Nazikian, J. Modern Optics **44**, 1037 (1997)]

Optical Imaging



Variation of mean intensity and intensity fluctuations along the optical axis for fluctuation levels (%) 0.1 (black), 0.5 (blue), 1.0 (green), 1.5 (red), 2.0 (magenta), 2.5 (cyan)



Synthetic Imaging – Phase Screen Model

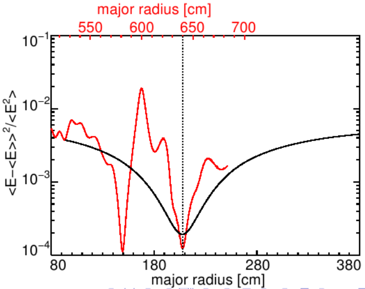
- Assume phase shift $\delta\phi(y - vt) << 1$ is imposed on field $E(x)\exp(-i\omega t)$ at $x = 0$.
- At $x = L$, each Fourier component k_y has a phase shift

$$\Delta\phi(k_y) = (k_x - k_0)L = k_0 L(\sqrt{1 - k_y^2/k_0^2} - 1) \simeq -\frac{k_y^2}{2k_0}L$$

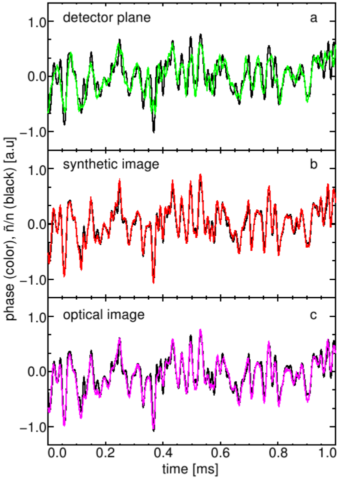
and a frequency shift $\Omega(k_y) = k_y v$.

- Apply a phase shift $\exp(i\Omega^2 L/2k_0 v^2)$ to each Fourier compononent $E(L, \omega + \Omega)$
- Practically, adjust L to minimize amplitude fluctuations

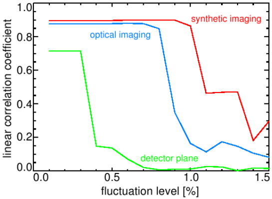
OPTICAL, SYNTHETIC, $\delta n/n = 10^{-3}$

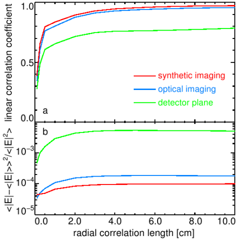


Both Techniques provide good fidelity at low fluctuation levels and long radial correlation lengths



$k_r = 0.2\ \text{cm}^{-1}$, $\delta n/n = 10^{-3}$

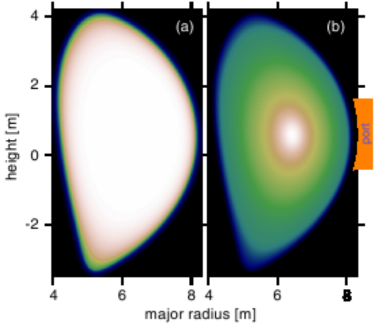


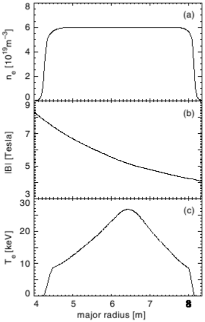


$L_r \equiv 2/k_r$

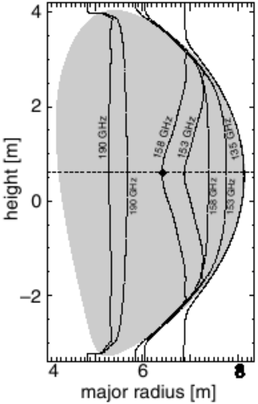
Demonstration that high T_e in ITER core importantly affects reflected field patttern (esp. upper X mode)

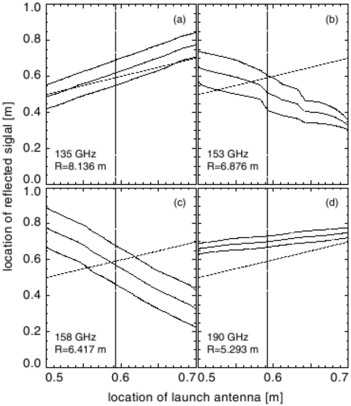
- For $T_e \sim 20$ keV, the relativistic mass shift leads to substantial movement of the reflection layer [Bindslev, Plasma Phys. Control. Fusion **35**, 1093 (1992)]





Variable curvature of the reflection surface with ω makes alignment of receiver with transmitter problematic





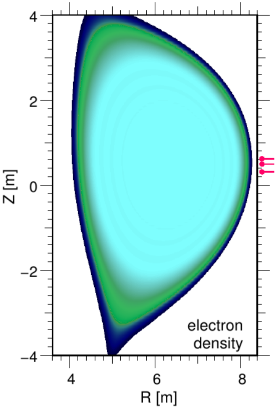
A combination of high gain transmitting and wide aperture receiving antennas may be necessary to ensure sufficient collection efficiency

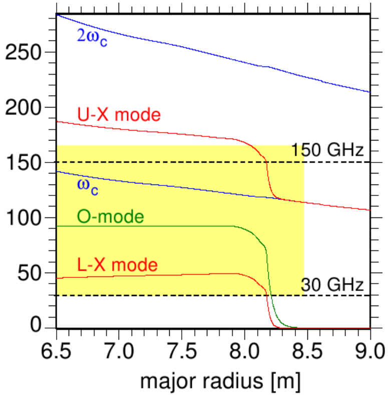
FWR2D has recently been extended to three dimensions (FWR3D)

- Both toroidal (vs 2D cylindrical) geometry and finite poloidal field are now included.
- The *vector* wave equation is solved in the full wave region. Required parallelization to achieve acceptable throughput.
- Initial application to edge reflectometry in ITER H mode profile. [G. Kramer, *et al*, 12^{th} International Reflectometry Workshop, Julich, May 18-20, 2015]
  - ITER modeling needs motivated 3D development
  - 3D needed for reflected power estimates

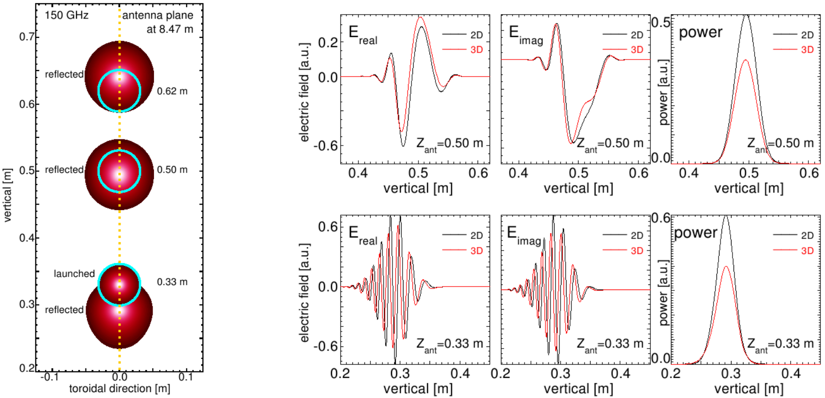
15MA ITER H-mode scenario profiles

Results from



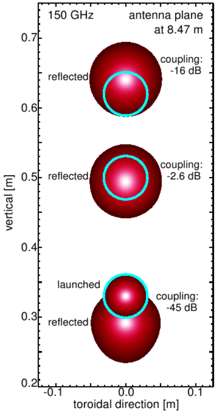


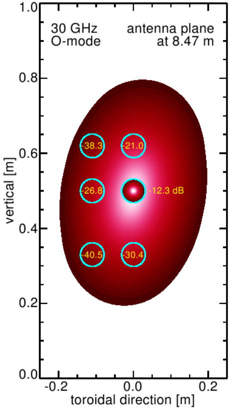
3D and 2D results agree well on symmetry plane $z = 0$



$B_p = 0$ for comparison

Reflected field patterns differ markedly for X and O modes

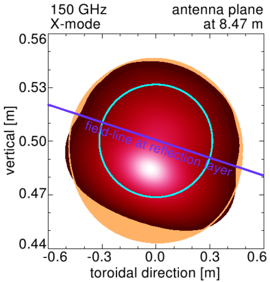


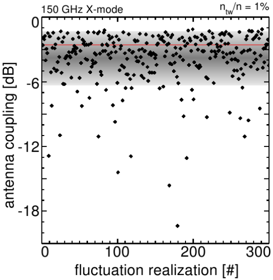


Reflected field pattern is rotated when $B_p \neq 0$. [P-A Gourdain and W. A. Peebles, Plasma Phys. Control. Fusion **50**, 025004 (2006)] *angle?*

Fluctuations introduce variance in coupling strength

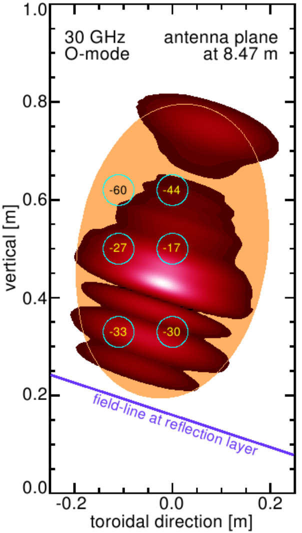
- 150 GHz X-mode
- Field aligned fluctuations, $k_r,\ k_z = 1\ \mathrm{cm}^{-1}\ \delta n/n = .01$
- An ensemble of 300 realizations yields average coupling of -3.8 db with 2.5 dB standard deviation vs -2.6 dB without fluctuations





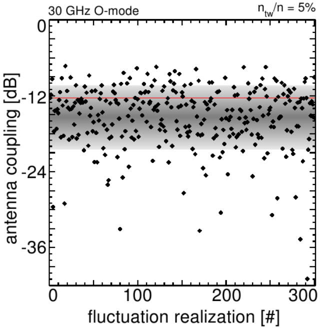
O mode coupling strength varies widely

Bi static coupling



Monostatic coupling

- 30 GHz O-mode
- Field aligned fluctuations, k_r, k_z $= 1\ \mathrm{cm}^{-1}$ $\delta n/n = .05$
- 300 realizations yields average coupling of -15.4 db with 5.0 dB standard deviation vs -12.4 dB without fluctuations



Ongoing Work and Plans

- Analyze core reflectometry in ITER
- Compare synthetic signals from XGC1 turbulence computations for NSTX with measured signals – Lei Shi's thesis.
- Quantify effects of diffraction and finite opacity on ECE imaging of low density edge plasma.
  - Requires inclusion of cyclotron absorption
- Interpretation of MIR signals in DIII-D
- Make FWR3D accessible. FWR2D is run as a service on the PPPL cluster, has a GUI (ELVIS).